Design of piled raft foundations by means of a multiphase model accounting for soil-pile interactions

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DESIGN OF PILED-RAFT FOUNDATIONS BY MEANS OF A MULTI-PHASE MODEL ACCOUNTING FOR SOIL-PILE INTERACTIONS

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ABSTRACT. *The settlement behavior of a pile-raft foundation is analyzed with the help of a multiphase model aimed at describing the overall behavior of the reinforced ground. According to this model, the group of piles is treated as a homogenized continuous medium in interaction with the soil along the pile length, as well as at their lower tips. These two kinds of interaction, which play a decisive role in the way the piles are actually working as strengthening elements, are described by specific constitutive laws which can be directly introduced in the general governing equations of the multiphase model. Thus improved, the multiphase model can then be incorporated in a finite element code developed in the context of an elastoplastic behavior of the soil, provided that the constitutive parameters of the interaction laws have been previously identified. This identification procedure is performed through numerical simulations of the simple problem of a single pile, loaded at its top. The global response of vertically loaded piled-raft foundations, expressed in the form of load settlement curves, is finally presented, clearly highlighting the decisive influence of the mobilized pile shaft friction and tip resistance on the foundation settlement reduction.*

1 INTRODUCTION

The setup of rational and reliable design methods for piled-raft foundations still remains a major computational challenge. Indeed, referring for instance to a finite element simulation of this kind of geotechnical structures, a fully three-dimensional analysis is required, with a locally refined mesh discretization in order to capture with sufficient accuracy the complex interactions prevailing between the piles, the surrounding soil and the raft. This leads to the elaboration of a complex and sophisticated computational tool, the use of which remains limited to rather simple configurations, such as purely vertical loading (Lee et al., 2010). Conceived as an improved homogenization procedure, a so-called “multiphase model” has been proposed, which can easily be implemented in a f.e.m.-based numerical code, thus leading to a considerable simplification of the initial design problem and to dramatically reduced computational times (Sudret & de Buhan, 2001).

The present contribution is focused on extending the range of applicability of the model, in order to account for soil-pile interactions taking place not only along the pile length (mobilized “shaft or skin friction”), as already developed in (Bourgeois et al., 2010), but also at the lower ends of the piles (mobilized “tip resistance”), both kinds of interactions being captured through specific laws expressed in the formalism of the multiphase model. The analysis is carried out following two successive steps. First, appropriate values for the constitutive stiffness and strength parameters of the above interaction laws are identified from numerical simulations performed on a representative volume of pile-reinforced ground. These parameters are then incorporated in the multiphase model and implemented in a finite element code, allowing to investigate the behavior of pile-reinforced foundations and thus provide insight into the way the foundation settlement is actually reduced by the incorporation of piles.

2 PRINCIPLE OF THE MULTIPHASE APPROACH

2.1 Problem statement

The typical engineering problem to be dealt with is that of a shallow strip footing of width B resting upon a soft clay which has been previously reinforced by a group of piles of length L placed just beneath the raft. The latter is subject to a purely vertical loading characterized by a linear density Q along the raft's axis Oz, as sketched in Figure 1(*a*). Denoting by ρ the pile's radius and by s the spacing between two adjacent piles assumed to be distributed in the soil mass following a regular square pattern, a key parameter of such a reinforcement scheme is the *reinforcement volume fraction* defined as:

$$\eta = \pi \frac{\rho^2}{s^2} \tag{1}$$

which is generally small. Our objective is to investigate through numerical simulations the actual settlement reduction provided by the piles.

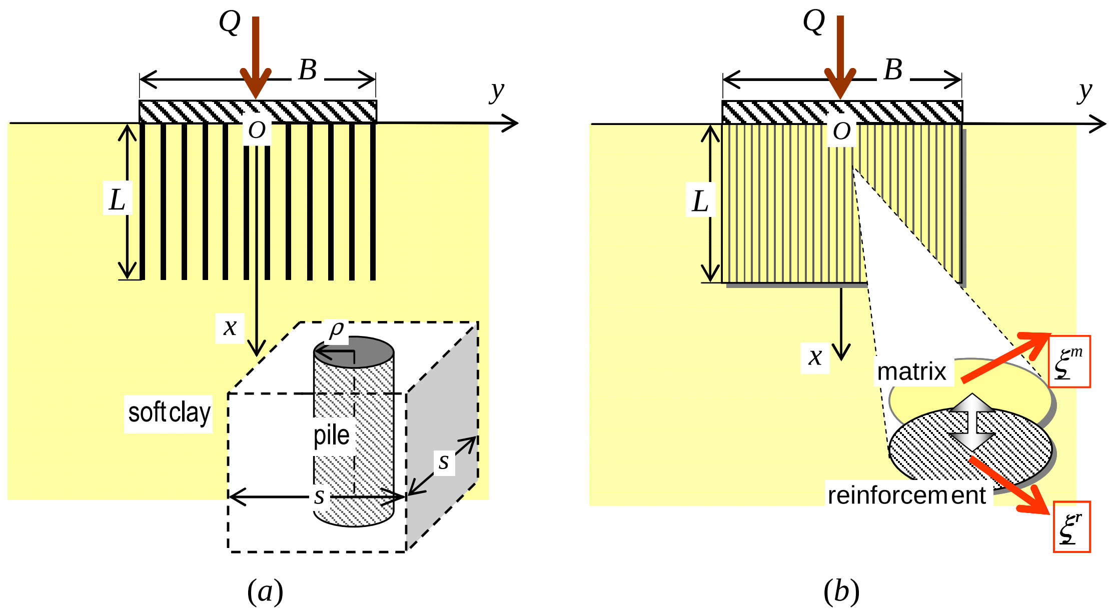

Fig. 1. Piled raft under vertical loading: (*a*) initial problem and (*b*) multiphase description

2.2 Outline of the multiphase model

The multiphase approach consists in replacing the composite pile-strengthened zone of height L and width B located beneath the raft, not by one single equivalent medium as in the traditional homogenization approach, but by *two superposed interacting continua*, called "phases". According to this model, a detailed presentation of which may be found in (Sudret & de Buhan, 2001), the soil is represented by a *matrix phase*, while the group of piles is represented by a *reinforcement phase*. More precisely, two coincident particles are located at any geometrical point of the reinforced zone; each particle is attributed its own kinematics, namely a displacement vector $\underline{\xi}^m$ for the matrix phase particle and $\underline{\xi}^r$ for the reinforcement phase particle.

The matrix phase is a classical continuous medium where the stress at any point is defined by a tensor $\underline{\underline{\sigma}}^m$, whereas the stress in the reinforcement phase occupying the domain V is defined by a uniaxial tensor $n^r \underline{e}_x \otimes \underline{e}_x$ along the vertical pile orientation, where n^r can be interpreted as the axial force in the piles per unit cross sectional area of reinforced soil. The equilibrium equations, expressed for each phase separately, may be written as:

$$\begin{aligned} &\mathrm{div}\underline{\underline{\sigma}}^m + I\underline{e}_x = 0 && \text{for the matrix phase} \\ &\mathrm{div}(n^r \underline{e}_x \otimes \underline{e}_x) - I\underline{e}_x = 0 && \text{for the reinforcement phase} \end{aligned} \tag{2}$$

where, for the sake of simplicity, the external body forces (gravity) have been omitted. In the above equations, I is a body force volume density, which represents, at the macroscopic scale of the multiphase model, the action of the piles on the soil along their length ("shaft or skin friction").

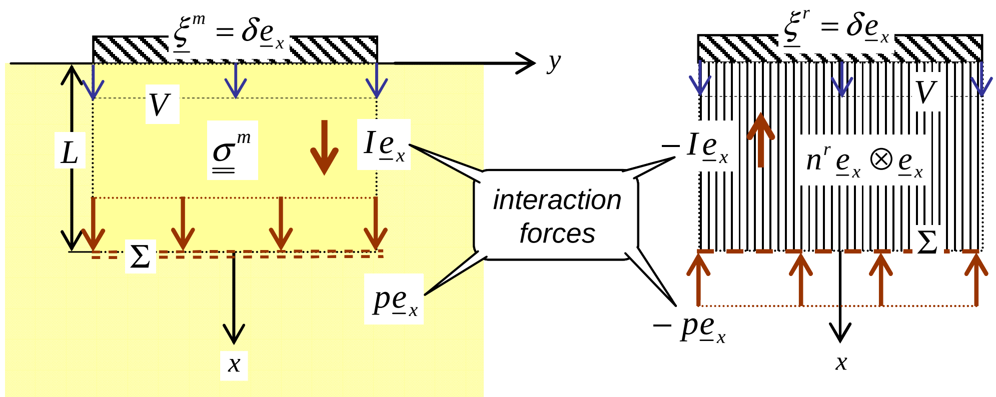

Fig. 2. Statics of the piled raft foundation modelled as a system made of two mutually interacting phases

A second kind of soil-pile interaction should be taken into account, corresponding to the action exerted by the pile lower tips onto the soil. This interaction takes place on the horizontal surface Σ located at a depth L from the surface, as shown in Figure 2, in the form of a surface density $p\underline{e}_x$, while the matrix phase exerts on the lower boundary surface Σ of the reinforcement phase an opposite surface density $-p\underline{e}_x$ (right hand side of Figure 2). The existence of such an interaction surface density, associated with the "pile tip resistance", implies the following condition on the lower boundary of the reinforcement phase:

$$n^r(x = L) = -p \tag{3a}$$

It generates at the same time in the matrix phase a *discontinuity of the vertical stress component* across Σ:

$$\sigma^m_{xx}(x = L^+) - \sigma^m_{xx}(x = L^-) = -p \tag{3b}$$

Besides, it should be noted that the same uniform displacement boundary condition is imposed by the rigid raft on top of both phases (Figure 2):

$$\underline{\xi}^m(x = 0) = \underline{\xi}^r(x = 0) = \delta\underline{e}_x \tag{4}$$

where δ is the foundation surface settlement.

2.3 Constitutive equations of the multiphase model

Assuming that the pile remain elastic, the constitutive equation of the reinforcement simply writes:

$$n^r = \alpha \varepsilon^r \tag{5}$$

where $\varepsilon^r = \partial \xi_x^r / \partial x$ is the *axial strain* of the reinforcement phase, while α is the *axial stiffness of the piles per unit transverse area*, which can be evaluated as the product of the reinforcement volume fraction by the Young's modulus E^p of the pile constituent material:

$$\alpha = \eta E^p \tag{6}$$

On the other hand the matrix phase constitutive relations are simply identified with those of the soil, modelled as a linear elastic perfectly plastic, purely cohesive material, with an cohesion C.

Furthermore, the above described soil-pile interactions are governed by specific constitutive laws which can be written as follows.

- The first kind of interaction ("shaft friction") is formulated by means of a relationship linking the interaction force *volume* density I to the relative axial displacement between the reinforcement and the matrix phases, defined as:

$$\Delta = \xi_x^r - \xi_x^m \tag{7}$$

  In the context of an elastic perfectly plastic behavior, such a constitutive law takes the following form:

$$I = c^I (\Delta - \Delta^p) \quad \text{with} \quad \dot{\Delta}^p = \begin{cases} 0 \text{ if } |I| \le I^0 \\ \ge 0 \text{ if } I = +I^0, \dot{I} = 0 \\ \le 0 \text{ if } I = -I^0, \dot{I} = 0 \end{cases} \tag{8}$$

  where Δ^p is the plastic component of the relative displacement, while c^I and I^0 are coefficients describing the stiffness of the interaction and the threshold value of the interaction force density for which an irreversible relative displacement between the matrix and reinforcement phases occurs. The latter parameter can for instance be related to the maximum skin friction between the piles and the ground (Bourgeois et al., 2010).

- Similarly, the second type of interaction, associated with the "pile tip resistance", will be expressed by a relation between the interaction force *surface* density p and the axial reinforcement/matrix relative displacement on $\Sigma(x=L)$. Adopting an elastoplastic framework, we thus obtain:

$$p = c^p \left(\Delta(L) - \Delta^p(L)\right) \quad \text{with} \quad \dot{\Delta}^p = \begin{cases} 0 \text{ if } |p| \le p^0 \\ \ge 0 \text{ if } p = +p^0, \dot{p} = 0 \\ \le 0 \text{ if } p = -p^0, \dot{p} = 0 \end{cases} \tag{9}$$

Both interaction constitutive laws are represented in Figure 3 below, in the form of classical stress-stress diagrams.

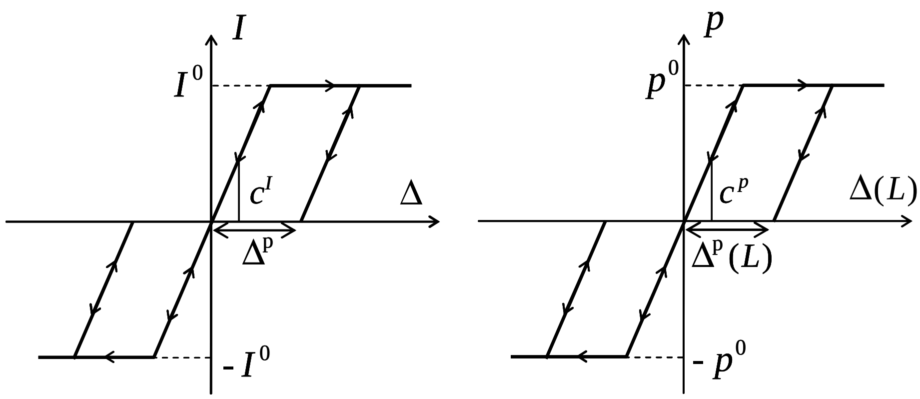

Fig. 3. Stress-strain diagrams for (*a*) the "shaft friction" and (*b*) "tip resistance" interaction laws

These interaction constitutive laws are strongly reminiscent of load-transfer curves ("*t-z*" curves) classically introduced for designing the load bearing capacity of individual piles driven in a soil (see among the most recent references: Ashour et al., 2010). It should be emphasized that an important difference between the above interaction laws and the usual load-transfer curves, is that the relevant kinematic variable associated with the pile-ground interaction forces is the relative displacement (Δ) between the matrix and reinforcement phases and not the absolute settlement (z) of the pile.

3 IDENTIFYING THE CONSTITUIVE PARAMETERS OF THE MULTIPHASE MODEL

The key ingredient to the application of the multiphase approach to the simulation of the piled-raft settlement behavior lies in the identification of the different constitutive parameters introduced above.

3.1 Matrix and reinforcement phases

The identification of the constitutive stiffness and yield strength parameters of both phases is very straightforward. Owing to the fact that the reinforcement volume fraction introduced in Eq. (1) is small, and consequently the soil volume fraction is close to unity, the matrix phase is assigned the same elastoplastic characteristics as the purely cohesive soft clayey soil, namely:

$$E^m = 45\,\text{MPa}\,,\ \nu^m = 0.3,\ \ C^m = 30\,\text{kPa} \tag{10}$$

As regards the reinforcement phase, assumed to remain elastic as the (concrete) piles, the axial elastic stiffness density is simply calculated from Eqs. (1) and (6). Thus:

$$\begin{aligned} &\rho = 0.25\text{m}, s = 2\text{m} \rightarrow \eta = 4.9\% \\ &E^p = 12500\,\text{MPa} \rightarrow \alpha = 613\,\text{MPa} \end{aligned} \tag{11}$$

3.2. Elastic interaction parameters

The determination of the interaction stiffness parameters c^I and c^p is based upon the same procedure as that used for piled-embankments (Hassen et al., 2009), which can be briefly described as follows. Considering the representative elementary volume of reinforced ground comprising one single pile surrounded by the soil, subject to a laterally constrained compressive loading, as pictured in Figure 4, the numerical simulation of this auxiliary problem (performed by means of a standard finite element code) is compared with the solution derived from the multiphase description of the same problem, which may be expressed analytically in the context of linear elasticity.

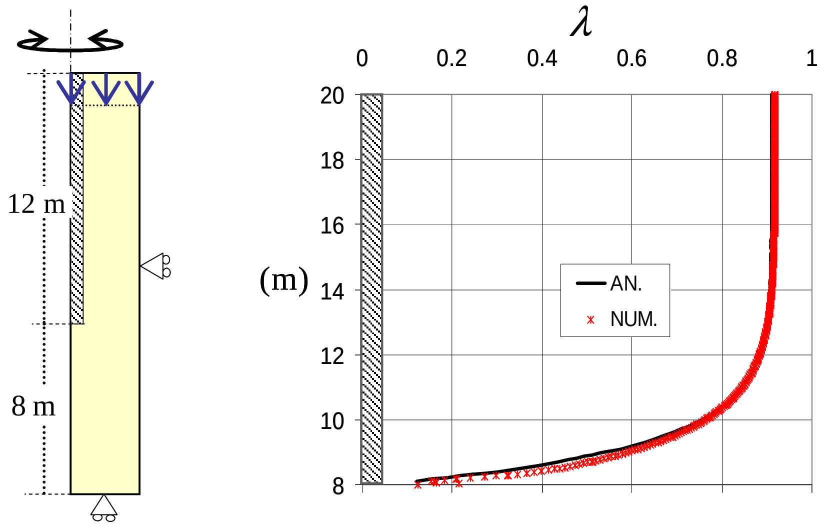

Fig. 4. Identification of the interaction stiffness parameters

More precisely, the identification is realized from fitting the numerical and analytical curves giving the variation with depth of the proportion λ of the total loading supported by the reinforcing pile. In the present configuration, the values of the interaction coefficients associated with the best fitting of these curves (Figure 4) are:

$$c^I = 40\,\text{MPa.m}^{-2},\ \ c^p = 5\,\text{MPa.m}^{-1} \tag{12}$$

3.2. Yield strength interaction parameters

The parameters governing the strength of the interaction laws (i.e. I^0 and p^0) can also be very simply evaluated on the basis of the numerical solution to the auxiliary problem, the loading being applied up to obtaining a complete plastification of the soil surrounding the pile. Figure 5 displays the corresponding axial load distribution along the pile length, expressed in terms of (compressive) stress n^r in the reinforcement phase, which turns out to vary linearly from -204 kPa at the pile head (x=0m) to -32 kPa at the pile tip (x=12m).

Such a result can be interpreted as follows in the framework of the multiphase model, assuming that both interaction forces have reached their yield values:

$$I = I^0 \ \text{ and } \ p = p^0 \tag{13}$$

Indeed, it follows immediately from combining the above equalities with the equilibrium equation of the reinforcement phase (second equation in (2)), along with the boundary condition (3a) that:

$$n^r(x) = \overbrace{n^r(L)}^{-p^0} - I^0(L-x) \tag{14}$$

so that:

$$p^0 = -n^r(L) = 32\,\text{kPa} \ \text{ and } \ I^0 = \frac{n^r(L) - n^r(0)}{L} = 14.33\,\text{kPa.m}^{-1} \tag{15}$$

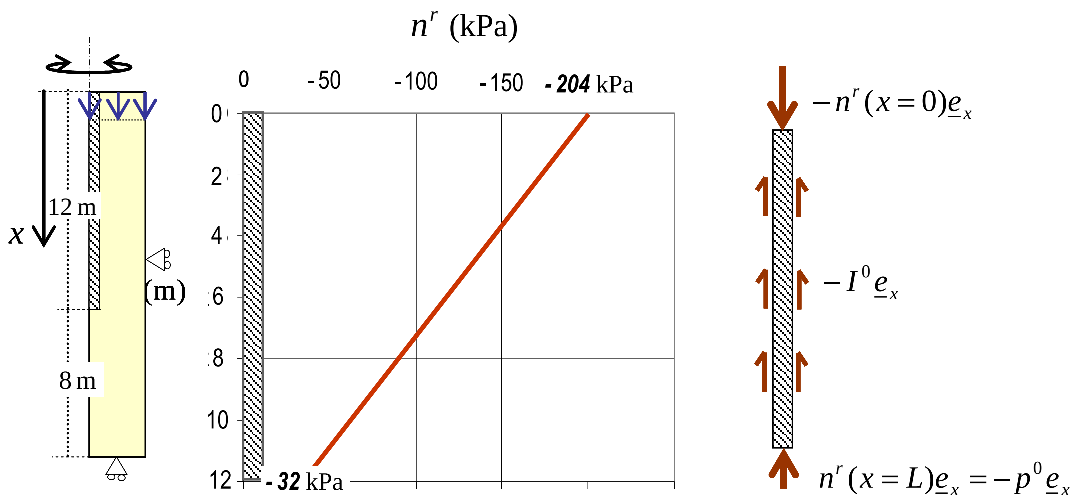

Fig. 5. Axial load distribution along the reinforcement for a fully plastic surrounding soil in the auxiliary problem

4 EXAMPLE OF NUMERICAL SIMULATION

Due to the symmetry with respect to the vertical plane passing through the footing axis, only one half of the piled raft foundation to be analyzed as a plane-strain problem, is represented in Figure 6, with its corresponding finite element mesh of 1,732 triangular elements and 3,542 nodes. The total width of the rigid footing acting on top of the pile-reinforced is equal to 12 m, so that the reinforcement scheme consists in six rows of 12m piles regularly spaced by a distance of 2m. All the other geometrical and geotechnical characteristics adopted in the following simulations are the same as those previously introduced.

It should be pointed out that the refinement of the mesh in the reinforced zone is the same as for a non reinforced, and thus homogeneous, soil. The only significant difference lies in the fact that three degrees of freedom (instead of two in the case of homogeneous soil) are attached to each node, namely the two components of the matrix phase displacement along with the axial relative displacement between phases Δ.

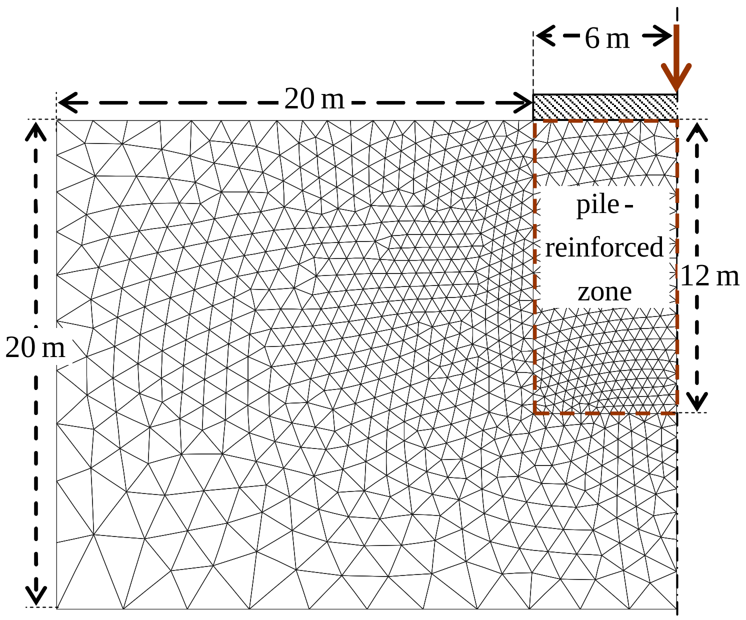

Fig. 6. Finite element discretization of the piled-raft foundation

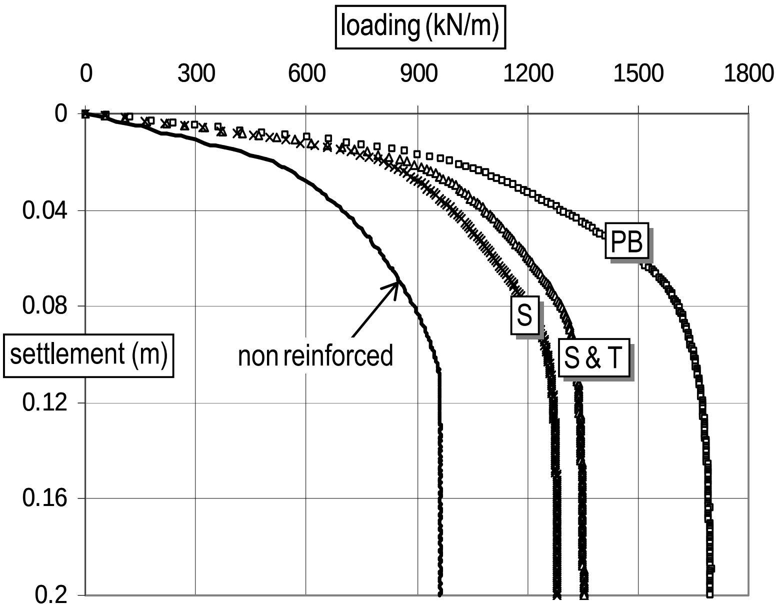

Fig. 7. Load-settlement curves of the piled and unpiled-raft foundation under different assumptions

The numerically computed load-settlement curves are displayed in Figure 7. The four curves drawn up to the ultimate bearing capacity of the structure (vertical branch of the curves) correspond to the following four different situations.

- First, the foundation without any pile (*non reinforced case*) for which the ultimate bearing capacity is equal to 960 kN/m, that is slightly above the classical value predicted by the Prandtl's failure mechanism ($(\pi+2)CB/2 \cong 930$ kN/m).
- The case of the piled-raft foundation, where the condition of *perfect bonding* ($\boxed{\text{PB}}$) between phases has been assumed, which can be enforced by assigning very high values to the interaction stiffness (c^I, c^p) and strength (I^0, p^0) parameters. The corresponding ultimate bearing capacity is equal to almost twice that of the un-piled raft: 1700 kN/m.
- The two remaining cases considered in the analysis are those where only the "side" interaction law is taken into account ($\boxed{\text{S}}$) and when both "side" and "tip" interactions are accounted for ($\boxed{\text{S\&T}}$) The corresponding ultimate bearing capacity is equal to 1350 kN/m in the latter case, 1280 kN/m in the former one.

Figure 8 represents a detailed picture of the load-settlement curves in the vicinity of a working load level of 1200kN/m, which exceeds the ultimate bearing capacity of the non reinforced foundation. Notable differences can be observed for the corresponding settlements evaluated on the basis of the above mentioned hypotheses: 3.3cm for the perfect bonding assumption, 8.1cm and 6cm for the "side interaction only" and "side and tip interaction" assumptions, respectively.

This influence is confirmed in Figure 9, showing the corresponding variations of forces along the pile located in the middle of the reinforce zone, expressed in terms of stress distributions n^r along the corresponding vertical line in the reinforcement phase. The most striking difference between the three profiles is the value of the reinforcement stress at the tip of the piles (x=12m), which vanishes when no tip interaction is taken into account, takes the maximum value of -127kPa when perfect bonding is assumed and the intermediate value of -31kPa (that is close to $-p^0$) when tip interaction is also considered in the analysis.

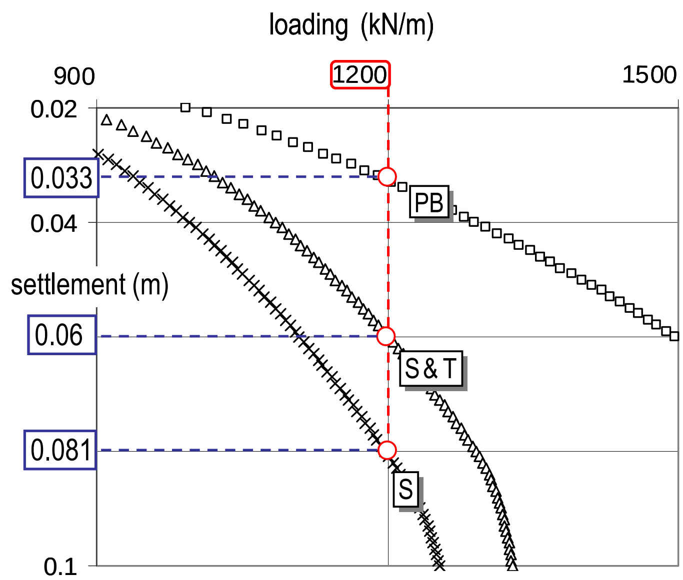

Fig. 8. Load-settlement curves of the piled and unpiled-raft foundation under different assumptions

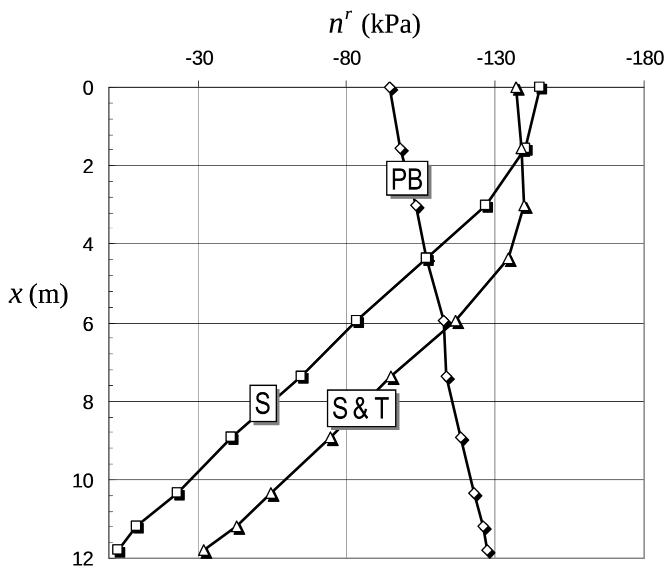

Fig. 8. Force distributions along the central pile for different interaction laws

5 CONCLUSION

It has been shown in this contribution how it was possible to incorporate specific soil-pile interaction laws in a multiphase model developed for the numerical simulation of piled-raft foundations. The important role played by such so-called “side and tip” interaction laws on the ultimate bearing capacity of the foundation, as well as on its settlement under working load conditions, has been clearly assessed in the analysis performed above.

Reliable predictions of piled-raft behaviour are therefore strongly dependent on the possibility of identifying appropriate values for the stiffness and yield strength interaction parameters to be introduced in the multiphase numerical calculations. The identification procedure proposed in this contribution, based on the numerical simulation of an auxiliary problem, should be used in a more systematic way, in order to produce closed-formed expressions giving the interaction parameters as functions of geometric (pile diameter and spacing) as well as constitutive soil and pile parameters, as it has been already done in the context of a linear elastic behaviour (Cartiaux et al., 2007).

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